Comment on "Nucleon form factors and a nonpointlike diquark"

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Abstract

Authors of Phys. Rev. C **60**, 062201 (1999) presented a calculation of the electromagnetic form factors of the nucleon using a diquark ansatz in the relativistic three-quark Faddeev equations. In this Comment it is pointed out that the calculations of these form factors stem from a three-quark bound state current that contains overcounted contributions. The corrected expression for the three-quark bound state current is derived.

The proper way to include an external photon into a few-body system of strongly interacting particles described by integral equations has recently been discussed in detail [1,2]. In particular, it has been shown how to avoid the overcounting problems that tend to plague four-dimensional approaches [1]. The purpose of this Comment is to point out that just this type of overcounting is present in the work of Bloch *et al.* [3] who calculated the electromagnetic current of the nucleon (and hence form factors), using the diquark ansatz in a four-dimensional Faddeev integral equation description of a three-quark system. Moreover, it is shown that the correct expression for the electromagnetic current consists of just three of the five contributions calculated in Ref. [3].

We begin by following Ref. [2] which is devoted to the discussion of the electromagnetic current of three identical particles, and is therefore directly applicable to the present case of a three-quark system. There we used the gauging of equations method to show that the bound state electromagnetic current of three identical particles is given by

$$j^\mu = \bar{\Psi}\Gamma^\mu\Psi \tag{1}$$

where Ψ ($\bar{\Psi}$) is the wave function of the initial (final) three-body bound state, and Γ^μ is the three-particle electromagnetic vertex function given by

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$$\Gamma^\mu = \frac{1}{6}\sum_{i=1}^{3}\left(\Gamma_i^\mu D_{0i}^{-1} + \frac{1}{2}v_i^\mu d_i^{-1} - \frac{1}{2}v_i\Gamma_i^\mu\right). \tag{2}$$

Here Γ_i^μ is the electromagnetic vertex function of the i'th particle, d_i is the propagator of particle i, v_i is the two-body potential between particles j and k (ijk is a cyclic permutation of 123), v_i^μ is the five-point function resulting from the gauging of v_i, and $D_{0i} \equiv d_j d_k$ is the free propagator of particles j and k. Because the bound state wave function Ψ is fully antisymmetric, we can write

$$j^\mu = \frac{1}{2}\bar{\Psi}\left(\Gamma_3^\mu D_{03}^{-1} + \frac{1}{2}v_3^\mu d_3^{-1} - \frac{1}{2}v_3\Gamma_3^\mu\right)\Psi. \tag{3}$$

The second term on the right hand side (RHS) of this expression defines the two-body interaction current contribution

$$j^\mu_{\text{two-body}} = \frac{1}{4}\bar{\Psi}v_3^\mu d_3^{-1}\Psi, \tag{4}$$

while the first and third terms together make up the one-body current contribution to the bound state current. As discussed in Ref. [1], the first term on the RHS of Eq. (3) defines an electromagnetic current

$$j^\mu_{\text{overcount}} = \frac{1}{2}\bar{\Psi}\Gamma_3^\mu D_{03}^{-1}\Psi \tag{5}$$

which overcounts the one-body current contributions, while the third term defines a current

$$j^\mu_{\text{subtract}} = \frac{1}{4}\bar{\Psi}v_3\Gamma_3^\mu\Psi \tag{6}$$

which plays the role of a subtraction term in that it removes the overcounted contributions. Here we shall not be concerned with the two-body interaction current, but rather, endeavour to examine the cancellations taking place between the first ("overcount") and last ("subtract") terms in detail. Thus we stress that the correct one-body contribution to the current, also known as the impulse approximation, is given by

$$j^\mu_{\text{impulse}} = j^\mu_{\text{overcount}} - j^\mu_{\text{subtract}}. \tag{7}$$

To reveal these cancellations one writes the bound state wave function in terms of its Faddeev components

$$\Psi = \Psi_1 + \Psi_2 + \Psi_3 \tag{8}$$

where

$$\Psi_i = \frac{1}{2}D_{0i}v_i\Psi. \tag{9}$$

These components are related through the Faddeev equations

$$\Psi_i = \frac{1}{2}D_{0i}t_i(\Psi_j + \Psi_k) \tag{10}$$

where t_i is the t matrix for the j-k system, and for identical fermions obey the symmetry relations [2]

$$P_{12}\Psi_1 = -\Psi_2, \qquad P_{13}\Psi_1 = -\Psi_3, \qquad P_{23}\Psi_1 = -\Psi_1, \qquad \text{etc.} \tag{11}$$

where P_{ij} is the operator interchanging particles i and j. The term with overcounting is thus

$$j^\mu_{\text{overcount}} = \frac{1}{2}\left(\bar{\Psi}_1 + \bar{\Psi}_2 + \bar{\Psi}_3\right)\Gamma^\mu_3 D^{-1}_{03}\left(\Psi_1 + \Psi_2 + \Psi_3\right) \tag{12}$$

which after the use of Eqs. (11) becomes a sum of five terms

$$\begin{aligned} j^\mu_{\text{overcount}} &= \frac{1}{2}\bar{\Psi}_3\Gamma^\mu_3 D^{-1}_{03}\Psi_3 + \bar{\Psi}_3\Gamma^\mu_1 D^{-1}_{01}\Psi_3 \\ &+ \bar{\Psi}_2\Gamma^\mu_1 D^{-1}_{01}\Psi_3 + \bar{\Psi}_2\Gamma^\mu_2 D^{-1}_{02}\Psi_3 + \bar{\Psi}_2\Gamma^\mu_3 D^{-1}_{03}\Psi_3. \end{aligned} \tag{13}$$

The diquark ansatz used in Ref. [3] is equivalent to invoking the separable approximation for the two-body t matrix:

$$t_i = h_i\tau_i\bar{h}_i, \tag{14}$$

with τ_i playing the role of the diquark propagator and h_i describing the vertex between the diquark and two free quarks. In the case of separable interactions, it is usual to define the spectator-quasiparticle (quark-diquark) amplitude X_i through the equation [1]

$$\Psi_i = G_0 h_i\tau_i X_i \tag{15}$$

where $G_0 = d_1d_2d_3$. In terms of these amplitudes the contribution of Eq. (13) becomes

$$\begin{aligned} j^\mu_{\text{overcount}} &= \frac{1}{2}\bar{X}_3 d_3\Gamma^\mu_3 d_3\tau_3\left(\bar{h}_3 d_1 d_2 h_3\right)\tau_3 X_3 + \bar{X}_3\tau_3\left(\bar{h}_3 d_1\Gamma^\mu_1 d_1 d_2 h_3\right) d_3\tau_3 X_3 \\ &+ \bar{X}_2\tau_2 d_2\bar{h}_2 d_1\Gamma^\mu_1 d_1 h_3 d_3\tau_3 X_3 + \bar{X}_2\tau_2 d_2\bar{h}_2\Gamma^\mu_2 d_2 d_1 h_3 d_3\tau_3 X_3 \\ &+ \bar{X}_2\tau_2 d_2\bar{h}_2 d_3\Gamma^\mu_3 d_1 h_3 d_3\tau_3 X_3. \end{aligned} \tag{16}$$

The five terms summed on the RHS of Eq. (16) are illustrated in Fig. 1. The last four terms are identical to the contributions $2\Lambda^i_\mu$ ($i = 2, \ldots, 5$) of Ref. [3], while the first term on the RHS of Eq. (16) differs from Λ^1_μ only in that our diquark propagator contains a dressing bubble. With or without this bubble, Eq. (16) does not give the correct impulse approximation.

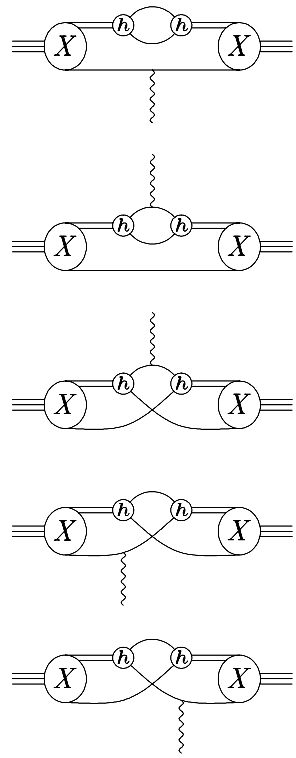

FIG. 1. Illustration of the five terms making up $j^\mu_{\rm overcount}$ in the case of separable interactions, Eq. (16). In the case of a three-quark system, a single line corresponds to a quark propagator d_i, a double line corresponds to the diquark propagator τ_i, a triple line corresponds to the three-quark bound state (the nucleon), and the wiggly line indicates the single-quark electromagnetic current Γ^μ_i. The correct impulse approximation to the three-body bound state current is obtained by removing the first (top) and fourth (second from the bottom) of these diagrams.

With the help of Eq. (9), the subtraction term of Eq. (6) can be expressed as

$$\begin{aligned} j^\mu_{\rm subtract} &= \frac{1}{2}\sum_{i=1}^{3}\bar\Psi_3 D_{03}^{-1}\Gamma_3^\mu\Psi_i \\ &= \frac{1}{2}\bar\Psi_3 D_{03}^{-1}\Gamma_3^\mu\Psi_3 + \bar\Psi_2 D_{02}^{-1}\Gamma_2^\mu\Psi_3. \end{aligned} \tag{17}$$

Comparison with Eq. (13) shows that the first and fourth terms of Eq. (13) are overcounted.[1] Thus the correct expression for the impulse approximation is

$$j^\mu_{\rm impulse} = \bar\Psi_3\Gamma_1^\mu D_{01}^{-1}\Psi_3 + \bar\Psi_2\Gamma_1^\mu D_{01}^{-1}\Psi_3 + \bar\Psi_2\Gamma_3^\mu D_{03}^{-1}\Psi_3. \tag{18}$$

For the work of Ref. [3], this means that the correct impulse approximation is given by the sum of their Λ^2_μ, Λ^3_μ, and Λ^5_μ only, and not, as claimed in their work, by the sum of all five

[1] Actually the fourth and fifth terms of Eq. (13) are identical, as can easily be shown using Eq. (19). Thus, although we have singled out the fourth term as the one being overcounted, it should be understood that overcounting is due to *either* the fourth or fifth terms.

Λ_μ^i's. Diagrammatically this means that the correct impulse approximation to the nucleon current in the diquark model corresponds to the sum of the second, third, and fifth diagrams of Fig. 1.

A further comment regarding Ref. [3] concerns the numerical values obtained for the contributions Λ_μ^1 and Λ_μ^5. By using the symmetry properties of Eqs. (11), one can rewrite the Faddeev equations, Eqs. (10), as

$$\Psi_i = D_{0i} t_i \Psi_j \tag{19}$$

where $i \neq j$. For separable interactions this implies that the amplitudes X_i satisfy the equations

$$X_i = \bar{h}_i D_{0i} h_j \tau_j X_j \tag{20}$$

where $i \neq j$. Using the time-reversed version of these equations one obtains $\bar{X}_3 = \bar{X}_2 \tau_2 \bar{h}_2 d_1 d_2 h_3$ which can be used to simplify the last term of Eq. (16):

$$\bar{X}_2 \tau_2 d_2 \bar{h}_2 d_3 \Gamma_3^\mu d_1 h_3 d_3 \tau_3 X_3 = \bar{X}_3 d_3 \Gamma_3^\mu d_3 \tau_3 X_3. \tag{21}$$

The RHS of this equation is just $2\Lambda_\mu^1$ of Ref. [3] and we have therefore shown that

$$\Lambda_\mu^1 = \Lambda_\mu^5. \tag{22}$$

This equality appears not to be reflected in the numerical results of Ref. [3] as is evident from their Table II.

Finally, we note that the errors of Ref. [3] have been perpetuated in a recent preprint [4].

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